

Counting

You have a 5 gallon jug and and 3 gallon jug and unlimited water.

You can fill a jug to capacity or empty it completely.

Use these two jugs to measure exactly 4 gallons.



You and your friend have a full 8 oz bottle of water and empty 3 oz and 5 oz bottles.

By either filling or emptying completely divide the 8 oz into two exactly equal amounts of 4 oz.

Three men robbed a gentleman of a vase containing 24 ounces of balsam.

Whilst running away they met a glass seller, of whom they purchased three vessels. On reaching a place of safety they wished to divide the booty, but found that their vessels could hold 5, 11, and 13 ounces respectively.

How could they divide the balsam into equal portions?

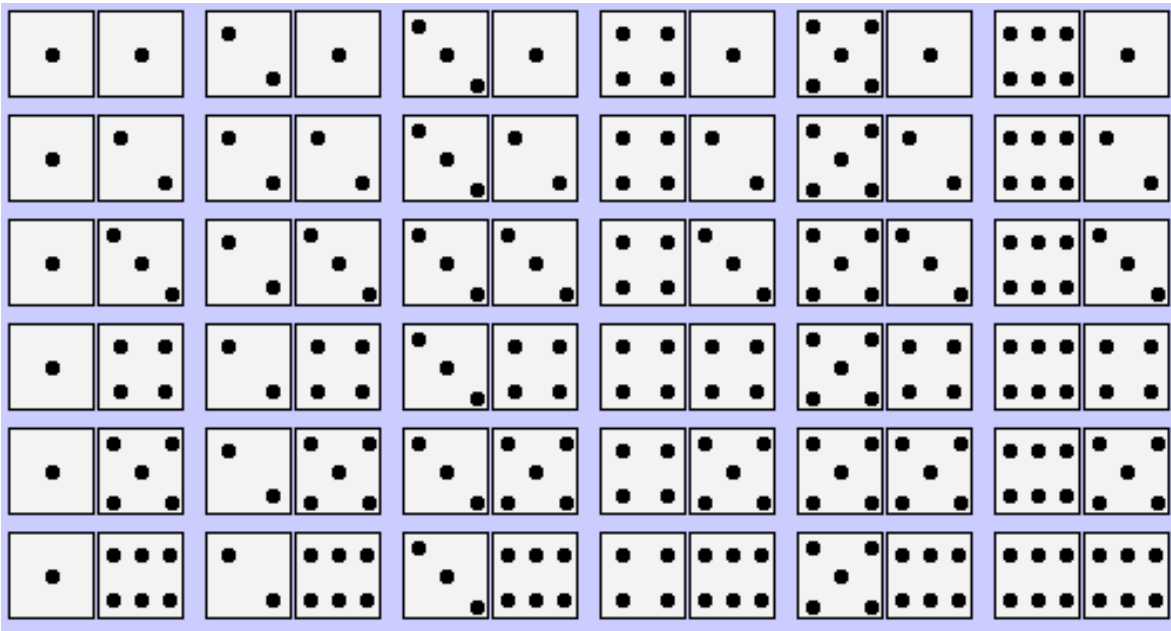
Adam, Bob, Claire and Dave are out walking: They come to a rickety old wooden bridge. The bridge is weak and only able to carry the weight of two of them at a time. Because they are in a rush and the light is fading they must cross in the minimum time possible and must carry a torch (flashlight,) on each crossing.

They only have one torch and it can't be thrown. Because of their different fitness levels and some minor injuries they can all cross at different speeds. Adam can cross in 1 minute, Bob in 2 minutes, Claire in 5 minutes and Dave in 10 minutes.

Adam, the brains of the group thinks for a moment and declares that the crossing can be completed in 17 minutes. There is no trick. How is this done?

How many possible outcomes are there for rolling two 6-sided dice?





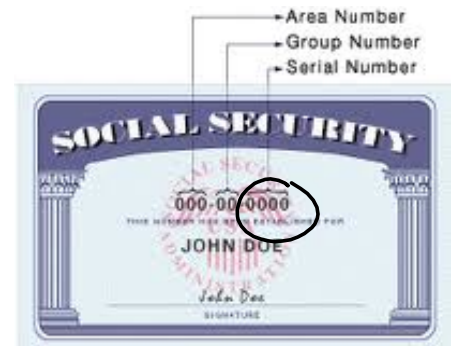
By List

{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) }

Another Table

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are four digits in the last section of your social security number. If all the digits 0 to 9 can be used, how many different arrangements are possible?



Say you want to go to the movies with a friend. You can walk, bike, or catch a bus to your friend's place, then you can walk, or catch a bus to the movie theater.

How many different ways can you get to the movie theater?



In your wardrobe you have 8 shirts, 3 pants, 12 pairs of socks, and 3 pairs of shoes. Assuming there's no combination you wouldn't wear and you always match your socks, how many different ensembles can you put together using your clothing collection?



At the local Home Depot you can buy paint.

It comes in the 8 colors:

Poppy, Sky, Cloud, Midnight, Latte, Forest, Orange, and Flame;

it can be **latex** or **oil based**;

with an **Eggshell, Flat, Semi-gloss, or High-gloss** finish;

and it's available for **indoor** or **outdoor** use.

How many different kinds of paint are there?



Oregon offers car owners the choice of a vanity license plate for their car. They use a mixture of letters and digits. The list below shows how they can be configured. It can use any of the 26 letters of the alphabet, any of the 10 digits 0 to 9, spaces, and hyphens.



Vanity: 1 to 6 letters, digits, spaces, or hyphens.

The hyphen cannot appear first or last.

No other characters are allowed.

How many possible vanity license plates are possible?

Permutations

Order matters: $ab \neq ba$

A window designer has been asked to arrange three pairs of the latest shoes in the front window of “The Well Heeled”.

How many different ways can the pairs of shoes be arranged in a line in the front window?



In your pocket you have four bills:

two \$1s, a \$2, and a \$5.

How many ways can you pull out two bills at random?

What possible amounts would you have?

On a ring type combination lock you have three rings, each with 10 digits on them. To unlock the combination you set the numbers on the three rings in a specific order.

How many possible codes are there?



Netflix offers these seven movies in their top scary picks:

- The Sixth Sense
- Sleepy Hollow
- The Babadook
- Corpse Bride
- A Girl Walks Home Alone At Night
- Rosemary's Baby
- Scream

Netflix asks that you rank these movies in order of your preference so that its AI can suggest other movies you may enjoy watching.

How many different ways can you rank these movies?

Suppose Netflix ask you to rank only five of the seven, how many ways can you do this?

The cheer team has 7 routines from which they can only choose 4 for a competition. Each routine can only be used once in the competition. How many possible arrangements of routines can the squad perform?



In horse racing, a trifecta is a bet picking the three winning horses in order of 1st, 2nd, and 3rd.

If there are ten horses in a race, how many ways are there to bet a trifecta?



In the Harry Potter Club the eight members decide to hold elections. There will be three positions open: **Wizardissimo**, **Scribus Maximus**, and **Galleon-ure**.

How many different ways can the eight members of the club be elected to the three positions?

If you have 12 tunes on your player and you set it to play only 7 tunes at random, how many different arrangements of seven tunes are possible? It won't repeat a tune and it will stop when all seven tunes have played.



How many ways can 3 cars and 4 trucks be driven away from a yard containing a selection of 8 unique cars and 11 unique trucks?

To win the Powerball[®] you have to correctly choose

5 numbers between 1 and 55,
AND
1 number between 1 and 42.

If you cannot repeat a number, how many possibilities are there?



So far we have only looked at ordering lists that have unique elements.

What happens when there are non-unique items in the list?



How many ways can you order 4 red and 3 green balls?
Remember the red balls are indistinguishable as are the green balls.

How many ways can you arrange the letters in the word
MATH?

How many ways can you arrange the letters in the word
POPPY?

How many ways can you arrange only four of the letters in the word POPPY

- a) if both O and Y are included?
- b) if only one of O or Y is included?
- c) there are no restrictions?

Why does part c) above have the same answer as the prior slide?

Combinations

Order does not matter: $ab = ba$



Urn

You are designing hats and want to have two colors on your newest creation. If you have four balls of wool each a different color to choose from, how many different hats of two colors are possible.



A bicycle shop owner has 12 unique bicycles in her showroom. How many different ways can she choose five of them to display in the shop window?



The call letters of a radio station must have 4 letters. The first letter must be a K or a W.

How many unique radio station call signs are possible if repetitions are not allowed?

If repetitions are allowed?

How many different ways can 9 trophies be arranged in single-file along a shelf?

In your sock drawer you have 7 black, 3 red, and 4 white individual socks.

If you grab two socks at random from the drawer, how many different pairs are possible?

In your sock drawer you have 7 black, 3 red, and 4 white individual socks.

If it's completely dark and you can't turn the lights on, what is the smallest number of socks you should grab to guarantee a matching pair?

At your weekly visit to Basket Robbers, the local ice cream parlor, you usually get 3 scoops of ice cream in a bowl.

There are 6 available flavors, so how many weeks will it take you to try every possible 3-scoop bowl?



Counting Worksheet

Fin

